8) NUMERICAL METHODS OF INTEGRATION

We use numerical approximations to find the area under a curve that is not able to be integrated, or that we do not have a function to integrate - just a model or diagram.

A) **Trapezium rule**

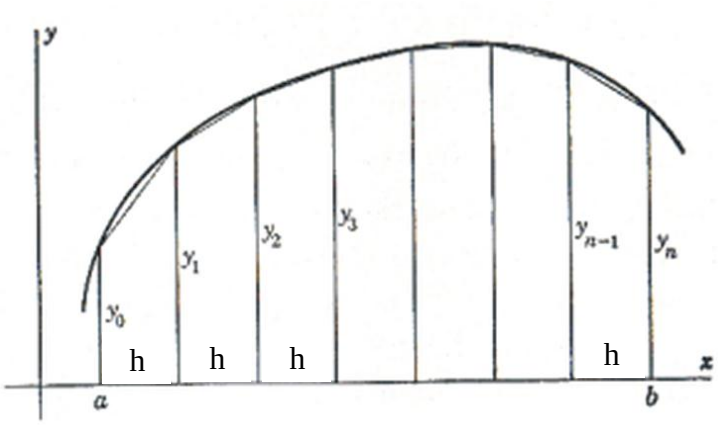
The area required is divided up into small sections that are approximated to the shape of trapeziums. The areas of all the trapeziums are added together to find a total **approximate** area.

In general, the trapezium rule is:

$$\text{Area} = \int_a^b f(x)\,dx$$

$$\approx \frac{h}{2}[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})]$$

y_0: 1st y-value; y_n: last y-value



where n = number of strips,
and h = width of each strip

Example: Use the trapezium rule to obtain estimate for the value of $\int_2^6 \frac{x^2}{x-1}\,dx$ using eight strips.

Set up a table of the y-values, use the TABLE function on the graphics calculator:

> Go to TABLE.
> Type in the equation $y = f(x)$.
> Press F5 for RANGE. Enter these values: Start = y_0, End = y_n, Pitch = h.

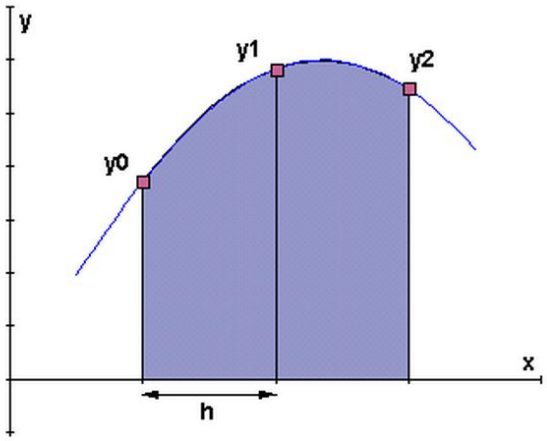
x	2	2.5	3	3.5	4	4.5	5	5.5	6
$y = \frac{x^2}{x-1}$	4	4.167	4.5	4.9	5.333	5.786	6.25	6.722	7.2

Substitute into the trapezium rule formula: $A \approx \frac{h}{2}[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})]$

$$A \approx \frac{0.5}{2}[4 + 7.2 + 2(4.167 + 4.5 + 4.9 + 5.333 + 5.786 + 6.25 + 6.722)]$$

$$= 21.6 \text{ units}^2 \text{ (1dp)}$$

B) **Simpson's Rule**



The Simpson's rule uses parts of a parabola to fit the shape of the curve. Each parabola passes through 3 points on the curve and has two regions below.
The number of y-values substituted into Simpson's Rule must be **odd**, and the number of strips must be **even**.

Simpson's Rule is:

$$A \approx \frac{h}{3}[y_0 + y_n + 4(y_1 + y_3 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2})]$$

All the even y-values — All the odd y-values

Example: Use Simpson's Rule to approximate the area under the curve $y = \frac{6}{x-2}$ between $x = 3$ and $x = 7$ using four strips.

x	3	4	5	6	7
$y = \frac{6}{x-2}$	6	3	2	1.5	1.2

$A \approx \frac{1}{3}[6 + 1.2 + 4(3 + 1.5) + 2(2)]$

$= 9.73$ units^2 (2dp)

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Delta Ex 22.3 pg 211 Q1 – 5